The results for the second family of solutions are qualitatively analogous, except that for them there are several stable zones (Fig. 4) and the characteristic increments of the most dangerous perturbations are smaller by almost an order of magnitude than in the case of the first family of solutions. These data give an answer to the problem of why preferentially highly linear wave elevations (almost to the extent of soliton waves) are observed in the experiments, particularly as Re increases.

NOTATION

ν, coefficient of kinematic viscosity; σ, surface tension coefficient; g, acceleration of free fall; ρ, liquid density; α, wave number; λ, wavelength; c, phase velocity of waves; $A_1 = h_{max} - h_{min}$; $We = \sigma\langle h\rangle/\rho\langle q\rangle^2$, Weber number; $Re = \langle q\rangle/\nu$, Reynolds number; $Fr = \langle q\rangle^2/g\langle h\rangle^2$, Froude number; Real (a), real part of the number a; $F_i = (\sigma/\rho)^3/g\nu^4$, film number; $A = 2.4\, d/d\xi(q_0/h_0) + z/h_0^2$; $B = 2.4\,(q_0/h_0) - c$; $p = 1.2\, d/d\xi(q_0^2/h_0^2) + F + 2zq_0/h_0^3 + 3\, d^3h_0/d\xi^3$; $D = 1.2\,(q_0^2/h_0^2)$; $V_{n-k+1+rN} = (A + i\alpha QB)_{n-k+1+rN} + i\alpha(k-1)B_{n-k+1+rN}$; $W_{n-k+1+rN} = (P + i\alpha QD - 3i\alpha^3Q^3h_0)_{n-k+1+rN} - i\alpha(k-1)(D - 9\alpha^2Q^2h_0)_{n-k+1+rN} + (9i\alpha^3Q(K-1)^2 + 3i\alpha^3(k-1)^3h_{0n-k+1+rN}$.

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STABILITY OF THE PROCESS OF EXTRUSION OF A VISCOELASTIC MATERIAL FROM A CONICAL CHANNEL

G. I. Burdé and T. M. Burdé

UDC 532.5:532.135

The stability of motion of a viscoelastic compressible medium in a conical channel with a small outlet orifice is investigated during the initial stage of extrusion.

The motion of viscoelastic media through conical channels is a process encountered in plastics production – solid-phase extrusion, fiber-forming, etc. In studies of these processes and their stability the steady-state motion of incompressible media is usually considered. At the same time, polymeric materials cannot be regarded as perfectly incompressible (see [1, 2]) and, clearly, in the initial stage of extrusion through a spinneret, when the exit velocity has not yet reached its steady-state value, volume compression of

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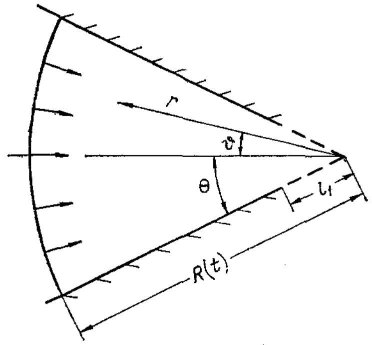

Fig. 1. Region of motion and coordinate system.

the material may occur. From the results of [3] it follows that the volume compression may be a cause of instability. This points to the need to take compressibility into account in investigating the stability of the process. We have therefore investigated the possibility of oscillatory instability and have determined the critical values of the extrusion rate and cone angle.

1. The basic motion, modeling the compaction process in the initial stage of extrusion and investigated for stability below, is taken in the form:

$$v_r^0 = U\frac{r}{R(t)};\ v_\vartheta^0 = v_\varphi^0 = 0. \tag{1}$$

Here, U is the constant extrusion rate, which is given on a certain moving spherical surface (Fig. 1), and R is the radius of that surface (U = dR/dt). The surface may be rigid (ram extrusion) of free (hydrostatic pressure extrusion). If we introduce a certain small conventional distance ℓ_1, where the conical part of the channel ends, then with the chosen kinematics the exit velocity will increase with time as $v = U\ell_1/R(t)$.

It is assumed that the material slides along the channel walls. In principle, this can be achieved by using a lubricant or material with a low coefficient of friction at the metal–polymer boundary. However, the choice of these kinematics may also be justified in other situations. Experimental investigations of the motion in spinneret inlets (see [4]) show that in most of the conical channel the streamlines are practically radial ($v_\vartheta \ll v_r$), and a shear velocity distribution is observed only in a narrow layer near the channel walls. Thus, (1) may be regarded as a model of the motion in the main channel, and the surface separating this zone from the shear layer may be regarded as the boundary.

The system of equations describing the isothermal motion of a compressible viscoelastic medium takes the form:

$$\frac{\partial\rho}{\partial t} + v_k\frac{\partial\rho}{\partial x_k} + \rho\frac{\partial v_k}{\partial x_k} = 0,$$

$$\rho\left(\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k}\right) = \frac{\partial\sigma_{ki}}{\partial x_k},$$

$$\sigma_{ij} = (K\varepsilon_{hk} + \mu e_{hk})\,\delta_{ij} + \tau_{ij}, \tag{2}$$

$$\frac{\partial\varepsilon_{ij}}{\partial t} + v_k\frac{\partial\varepsilon_{ij}}{\partial x_k} + \varepsilon_{kj}\frac{\partial v_k}{\partial x_i} + \varepsilon_{ik}\frac{\partial v_k}{\partial x_j} = e_{ij},$$

$$\lambda\frac{\delta_a\tau_{ij}}{\delta t} + \tau_{ij} = 2\eta\left(e_{ij} - \frac{1}{3}e_{hk}\delta_{ij}\right),$$

$$\frac{\delta_a\tau_{ij}}{\delta t} = \frac{\partial\tau_{ij}}{\partial t} + v_k\frac{\partial\tau_{ij}}{\partial x_k} - (\omega_{ik}\tau_{kj} - \tau_{ik}\omega_{kj}) + a\,(e_{ik}\tau_{kj} + \tau_{ik}e_{kj}),$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right);\ \omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right).$$

Here and in what follows, in all the relations the summation convention is assumed to apply.

The system of rheological relations (2) can be obtained on the basis of the approach developed in [5, 6], where the governing equations were constructed by the methods of the thermodynamics of irreversible processes with allowance for the relations between the reversible, irreversible and total strain rates. This approach leads to rheological equations of the type in question if it is assumed that the reversible strains are small as compared with the total strains, and that the irreversible volume changes are unimportant.

The motion (1) corresponds to the solution of system (2) with the uniform density, strain and stress distributions ρ^0, $\varepsilon^0{}_{ik}$, $\tau^0{}_{ik}$:

$$\tau_{ik}^0 = 0;\ \varepsilon_{ik}^0 = \frac{1}{2}\left(1 - \frac{R_0^2}{R^2}\right)\delta_{ik};\ \rho^0 = \rho_0\left(\frac{R_0}{R}\right)^3. \tag{3}$$

We will investigate the stability of the basic motion (1), (3) with respect to small perturbations. In Eqs. (2) we go over to the new set of independent variables t, $y_i = x_i/R$:

$$\frac{\partial}{\partial x_i} \to \frac{1}{R}\frac{\partial}{\partial y_i};\quad \frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \frac{U}{R} y_k \frac{\partial}{\partial y_k}$$

and represent the solutions of system (2) in the form:

$$v_i = Uy_i + u_i;\ \rho = \rho^0 + \Phi;\ \varepsilon_{ij} = \varepsilon_{ij}^0 + E_{ij},$$

where u_i, Φ, and E_{ij} are regarded as functions of the variables y_i and t. After linearization with respect to the perturbations, using (3) we arrive at the equations

$$\frac{\rho_0 R_0^3}{R^3}\left(\frac{\partial u_i}{\partial t} + \frac{U}{R} u_i\right) = \frac{K}{R}\frac{\partial E_{kk}}{\partial y_i} + \frac{\mu}{R^2}\frac{\partial^2 u_k}{\partial y_i \partial y_k} + \frac{1}{R}\frac{\partial \tau_{ki}}{\partial y_k}, \tag{4}$$

$$\frac{\partial E_{ij}}{\partial t} + \frac{2U}{R} E_{ij} = \frac{R_0^2}{2R^3}\left(\frac{\partial u_i}{\partial y_j} + \frac{\partial u_j}{\partial y_i}\right), \tag{5}$$

$$\tau_{ij} + \lambda\left(\frac{\partial \tau_{ij}}{\partial t} + \frac{2aU}{R}\tau_{ij}\right) = \frac{\eta}{R}\left(\frac{\partial u_i}{\partial y_j} + \frac{\partial u_j}{\partial y_i} - \frac{2}{3}\frac{\partial u_k}{\partial y_k}\delta_{ij}\right). \tag{6}$$

The solutions of Eqs. (4)-(6) must satisfy the boundary conditions on the surface of the cone and on the surface r = R(t). In what follows we will use the dimensionless radial coordinate $\xi = r/R$ and the spherical angular coordinates ϑ and φ. The condition of zero flow across the boundary of the cone gives

$$u_\vartheta = 0 \text{ when } \vartheta = \Theta. \tag{7}$$

On the surface r = R(ξ = 1) one of the following two conditions is given:

$$u_r = 0 \text{ when } \xi = 1, \tag{8}$$

$$e_{r\vartheta} = e_{r\varphi} = 0 \text{ when } \xi = 1, \tag{9}$$

where (8) is used in the case of a rigid boundary surface (ram extrusion) and (9) for a free surface (hydrostatic compression). Condition (9), which expresses the absence of shear stresses on the disturbed interface, is obtained by expanding all the quantities in a series in the surface perturbations near ξ = 1 with subsequent linearization.

The system (4)-(6) can be reduced to three equations for the quantities $\partial u_k/\partial y_k$, E_{kk} and $\partial^2\tau_{ki}/\partial y_k \partial y_i$. For this it is necessary to carry out convolution operations on Eq. (4) with the operator $\partial/\partial y_i$, on Eq. (6) with the operator $\partial^2/\partial y_i \partial y_j$, and in expression (5) with respect to the tensor incides. Going over to the dimensionless variables T, G, Q, and S defined below, we obtain

$$T = \frac{R_0}{R};\ G = \frac{1}{U}\frac{\partial u_k}{\partial y_k};\ Q = E_{kk};\ S = \frac{1}{\rho_0 U^2}\frac{\partial^2 \tau_{kl}}{\partial y_k \partial y_l}, \tag{10}$$

$$\frac{\partial G}{\partial T} - \frac{G}{T} = -\frac{D}{T^4}\Delta Q + \frac{M}{T^3}\Delta G - \frac{1}{T^4} S, \tag{11}$$

$$\frac{\partial Q}{\partial T}-\frac{2Q}{T}=-TG, \tag{12}$$

$$\frac{S}{T^2}+W\left(\frac{\partial S}{\partial T}-\frac{2aS}{T}\right)=-\frac{4}{3}\frac{B}{T}\Delta G. \tag{13}$$

$$D=\frac{K}{\rho_0 U^2},\ M=\frac{\mu}{\rho_0 R_0 |U|},\ B=\frac{\eta}{\rho_0 R_0 |U|},\ W=\frac{\lambda |U|}{R_0}. \tag{14}$$

The solutions of Eqs. (11)-(13) are represented in the form of a superposition of solutions with separable variables:

$$G=\sum_{n,l} g^{(n,l)}(t)\cdot f^{(n,l)}(\xi,\ \vartheta,\ \varphi);\ \Delta f^{(n,l)}=-h^2_{n,l}f^{(n,l)};$$
$$Q=\sum_{n,l} q^{(n,l)}(t)\cdot f^{(n,l)}(\xi,\ \vartheta,\ \varphi);\ S=\sum_{n,l} s^{(n,l)}(t)\cdot f^{(n,l)}(\xi,\ \vartheta,\ \varphi). \tag{15}$$

Using the known expressions for the eigenfunctions of the Laplace operator $f^{(n,\ell)}$ and the vector relation between $f^{(n,\ell)}$ and the velocity components $u_i^{(n,\ell)}$

$$f^{(n,l)}=\xi^{-1/2}J_{l+1/2}(h_{n,l}\xi)\,Y_l(\vartheta,\ \varphi),\quad u_i^{(n,l)}=-\frac{g^{(n,l)}}{h^2_{n,l}}\frac{\partial f^{(n,l)}}{\partial y_i}, \tag{16}$$

we can determine the values of $h_{n,\ell}$ from the boundary conditions. The two variants of the boundary condition for $\xi = 1$ (8), (9) give two variants of the relation determining $h_{n,\ell}$ for given ℓ:

$$hJ'_{l+1/2}(h)-\frac{1}{2}J_{l+1/2}(h)=0 \quad \text{(ram extrusion)} \tag{17}$$

$$hJ'_{l+1/2}(h)-\frac{3}{2}J_{l+1/2}(h)=0 \quad \text{(hydrostatic compression)}. \tag{18}$$

The no-flow boundary condition (7) gives the relation determining the possible values of ℓ:

$$\frac{dP_l^m(\cos\vartheta)}{d\vartheta}=0 \text{ when } \vartheta=\Theta. \tag{19}$$

As a result of the fact that the region of variation of the coordinate ϑ does not contain the point $\vartheta = \pi$, the index ℓ is not necessarily an integer and may take arbitrary values [7].

The equations for the amplitudes of the perturbations $g^{(n,\ell)}(t)$, $q^{(n,\ell)}(t)$, $s^{(n,\ell)}(t)$ can be obtained from (11)-(13) by means of the substitutions: $G \to g^{(n,\ell)}$, $Q \to q^{(n,\ell)}$, $S \to s^{(n,\ell)}$, $\Delta \to -h^2{}_{n,\ell}$. We will consider the solutions of these equations on the assumption that $D \gg 1$ (large bulk moduli, see (14)), which is well satisfied in the case of polymeric materials [1, 2]. The solutions can be found by the averaging method [8]. Considering that the solutions are oscillatory with frequency $\sim \sqrt{D}$ (see below), we go over from the quantities $q(n,\ell)$ and $s(n,\ell)$ to the variables z_2 and z_3, which are of the same order as the variable $g(n,\ell) = z_1$: from Eq. (11) it is clear that $s(n,\ell) \sim dg(n,\ell)/dT \sim \sqrt{D}g(n,\ell)$, $q(n,\ell) \sim g(n,\ell)/\sqrt{D}$. Introducing the "fast" time β, we obtain (in what follows we have dropped the indices from $h_{n,\ell}$):

$$\beta=\sqrt{D}T,\ z_1=g^{(n,l)},\ z_2=q^{(n,l)}\sqrt{D},\ z_3=\frac{s^{(n,l)}}{\sqrt{D}}; \tag{20}$$

$$\frac{dz_1}{d\beta}-\frac{h^2}{T^4}z_2+\frac{1}{T^4}z_3=\alpha\left(\frac{1}{T}-\frac{Mh^2}{T^3}\right)z_1, \tag{21}$$

$$\frac{dz_2}{d\beta}+Tz_1=\alpha\frac{2}{T}z_2, \tag{22}$$

$$\frac{dz_3}{d\beta}-\frac{Nh^2}{T}z_1=\alpha\left(\frac{2a}{T}-\frac{1}{WT^2}\right)z_3; \tag{23}$$

$$\alpha = \frac{1}{\sqrt{D}}, \quad N = \frac{4}{3}\frac{B}{DW}. \tag{24}$$

Here, the variable $T = \alpha/\beta$ plays the part of "slow" time [9].

In expressions (21)-(23) it is assumed that $W \sim 1$, $M \sim 1$, $N \sim 1$. Although from the last estimate there follows $B \sim D$ (see (24)), i.e., $\eta \gg \mu$, the dilatational viscosity must be taken into account. The results presented below show that even when $\mu \ll \eta$ the dilatational and shear viscosities have an equally important influence on the development of instability of the type in question. However, when $\mu \sim \eta$ ($B \sim M \sim 1$, the calculations are not given) the dilatational viscosity plays a decisive role.

In order to employ the averaging method the system (21)-(23) must be reduced to standard form [8]. The conversion formulas are determined by the form of the "generating" solution (the solution of the system (21)-(23) for $\alpha = 0$ and $T = \mathrm{const}$):

$$z_1 = A(\beta)\sin\psi(\beta), \quad z_2 = \frac{T}{\Omega}A(\beta)\cos\psi(\beta) + \frac{C(\beta)}{h^2},$$
$$z_3 = -\frac{Nh^2}{\Omega T}A(\beta)\cos\psi(\beta) + C(\beta); \quad \Omega = \left(\frac{h^2}{T^3} + \frac{Nh^2}{T^5}\right)^{1/2}. \tag{25}$$

Averaging the right sides of the equations in standard form with respect to the variable ψ and solving the averaged equations, we obtain

$$A = A_0 T\Omega^{1/2}\exp\left[\frac{Mh^2}{4T^2} + \frac{h^2N}{2}\int\frac{1}{\Omega^2}\left(\frac{2a}{T^6} - \frac{1}{WT^7}\right)dT\right],$$
$$\psi = \sqrt{D}\int\Omega(T)\,dT + \psi_0. \tag{26}$$

The conditions of growth of a given harmonic of the perturbation spectrum can be found from the inequality $dA/dT > 0$, which, applied to (26), gives for $T \approx 1$ (the initial stage of the process): $2h^2M(1 + N) < 1 - N(1 - 4a + 2/W)$. Using expressions (24) and (14), we can write this relation as an inequality for the velocity $|U|$ (the quantity N does not contain U):

$$|U| > \frac{2\mu(N+1)}{\rho_0 R_0[1 - N(1 - 4a)]}h^2 + \frac{NR_0}{\lambda[1 - N(1 - 4a)]}. \tag{27}$$

The instability condition (growth of at least one harmonic of the spectrum) is obtained from (27) when $h = h_{min}$.

The quantity h_{min} is defined as the root of Eq. (17) or (18) least in absolute value for the least ℓ satisfying condition (19). The set of values of ℓ permitted by relation (19) depends on the quantity Θ. For simplicity, we will do the calculations for the case $m = 0$ (axisymmetric perturbations). Using tables of zeros of the spherical functions P_ℓ' [10], it is possible for given Θ to determine the least value of ℓ satisfying (19), and for that ℓ find the first root of Eq. (17) or (18). Then, from relation (27) we determine the value of the velocity $|U|$ critical for instability. Since the least ℓ increases as Θ decreases, and the roots of (17) and (18) increase with ℓ, the critical value of the extrusion rate increases with decrease in the cone angle. Conversely, it is possible to determine the critical value of the angle Θ for a given $|U|$, by finding h^2 from (27) and determining the corresponding ℓ and Θ.

The results of the calculations are presented in Fig. 2, where each curve, corresponding to a certain value of λ, divides the plane (Θ, $|U|$) into regions of stability and instability (region of stability to the left of the curve). The curves in Fig. 2 were obtained for the case of ram extrusion (h is determined from Eq. (17)) for $a = -1$ (upper convective derivative in the rheological equation) and the following values of the parameters: $\rho_0 = 10^3$ kg/m³, $R_0 = 0.1$ m, $\mu = 0.1$ Pa·sec, $\eta/K = 0.075$ sec.

It is possible to obtain a simple expression for the dependence of the critical extrusion rate on the cone angle using asymptotic expressions [10, 11] for the roots of (17), (19):

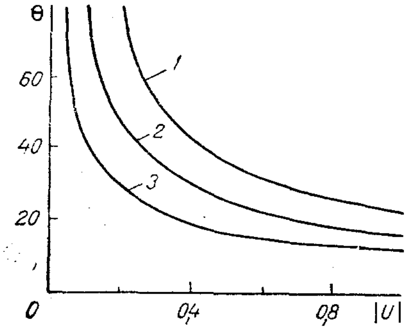

Fig. 2. Critical cone angle Θ (deg) as a function of the extrusion rate |U| (m/sec): 1) λ = 0.75 sec; 2) 1; 3) 5.

$$l+\frac{1}{2}=\frac{5\pi}{4\Theta},\ h_{l,1}=\left(l+\frac{1}{2}\right)+0{,}8086\left(l+\frac{1}{2}\right)^{1/3}. \tag{28}$$

Although Eqs. (28) were obtained on the assumption that $\ell \gg 1$, they give ℓ and $h_{\ell,1}$ quite accurately for all values of Θ of practical importance. Starting from $\ell \geq 5$ ($\Theta \leq 40°$ cone angle $\leq 80°$), the difference is not more than 2%. By combining expressions (27) and (28) it is possible to obtain a relation for the critical velocity.

Our investigations show that the initial stage of the process of extruding a viscoelastic material through a die or spinneret may be unstable. The oscillating nature of the instability is associated with the bulk elasticity of the material (see (26), (14)), and the growth of the oscillations is possible only under conditions of compaction of the material, when the velocity decreases in the direction of motion. Thus, the instability described does not have analogs among the various modes of instability typical of incompressible materials (see review [12]). For instability to occur it is not necessary to have a high degree of volume compression – if the extrusion rate is sufficient, the perturbations may grow from the very outset of compression. It should be noted, however, that the time during which motion with compaction takes place depends on the degree of compressibility (in the incompressible limit the corresponding time interval tends to zero). For low degrees of compressibility this time may be insufficient for instability to develop.

Unstable motion during the process of solid-phase poly mer extrusion was observed in the experiments reported in [13]. Defects in the extrudate were noted after a certain threshold ram velocity had been reached (defects of various types were observed, including spiral defects corresponding to $m \neq 0$). As the extrusion temperature was lowered, the threshold velocity fell, which, generally speaking, is in accordance with (27). In fact, a fall in temperature leads to an increase in the relaxation time and shear viscosity, so that N remains more or less constant, while the second term in (27) decreases. As noted by the authors of [13], the instability effects were not associated with friction at the boundary – the use of lubricants and materials with a low coefficient of friction at the metal–polymer boundary did not have a very significant effect on the defect formation patterns. All this makes it possible to assume that some of the modes of instability observed in [13] can be related to the type investigated in our study.

NOTATION

v^0_r, v^0_ϑ, and v^0_φ, velocity components of the basic motion in the spherical coordinate system; r, ϑ, and φ, spherical coordinates; R(t), distance to the ram or free surface; U, extrusion rate (velocity of ram or free surface); ℓ_1, a small conventional distance from the apex of the cone to the tip of the die (spinneret) cone; ρ, density; v_i, components of the total (basic motion plus perturbations) velocity in the Cartesian coordinate system; x_i, Cartesian coordinates; t, time; σ_{ij}, components of the stress tensor; τ_{ij}, deviatoric components of the stress tensor; ε_{ij}, Almansi strain tensor components; e_{ij}, strain rate tensor components; δ_{ij}, Kronecker delta; $\delta_a/\delta t$, Oldroyd derivative; a, derivative type index ($a = 0, -1$); ω_{ij}, rotation tensor components; K, bulk modulus; μ, dilatational viscosity coefficient; η, shear viscosity coefficient; λ, relaxation time; τ^0_{ij}, ε^0_{ij} and ρ^0, deviatoric stresses, strains and density for the basic motion; ρ_0 and R_0, density and the distance to the ram at the initial instant; u_i, velocity perturbation components; Φ, density perturbation; E_{ij}, perturbations of the strain tensor components; $\xi = r/R$, dimensionless radial coordinate that plays the part of Lagrangian coordinate for the basic motion; Θ, cone half-angle; $T = R_0/R$, a dimensionless independent variable that increases monotonically with time; G, Q, and S, dimensionless variables that determine the velocity, strain and shear stress perturbations respectively; D, M, B, and W, dimensionless parameters that determine the values of the bulk elasticity, the dilatational viscosity, the shear viscosity, and the relaxation time respectively; $\Delta = \partial^2/\partial y^2_k$, Laplacian; $f^{(n\ell)}$, eigenfunctions of the Laplacian; $h_{n,\ell}$, eigenvalues of the Laplacian determined from the boundary conditions (n is an index which numbers the values of h in increasing order, ℓ is the spherical harmonic index); $J_\nu(z)$, Bessel function of the first kind; $Y_\ell(\vartheta, \varphi)$, surface spherical harmonic; $P_\ell^m(\cos \vartheta)$, an associated Legendre function of the first kind; $g(n,\ell)$, $q(n,\ell)$, $s(n,\ell)$, time parts of the harmonics of the G, Q, S, value spectrum; z_1, z_2, and z_3, auxiliary dimensionless parameter; A, amplitude of the velocity perturbation fluctuations; ψ, phase of the fluctuations; A_0 and ψ_0, initial values of the amplitude and phase; $C(\beta)$, a variable corresponding to a monotonic mode; $\Omega(T)$, a variable that plays the part of frequency in the "generating" solution.

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RELAXATION OF CONCENTRATION INHOMOGENEITIES IN NONIDEAL SOLUTIONS

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Numerical solutions of the nonlinear diffusion equation are obtained for nonideal solutions satisfying an equation of state of the Van der Waals–Landau average field type. The results are compared with experiment.

The study of the processes of relaxation of concentration inhomogeneities in binary gaseous solutions in the region of states in which significant nonideality has a pronounced influence on the diffusion owing to the proximity of the critical line of the mixture is a complex problem involving the nonlinearity of the equations describing these processes. Particularly difficult to solve experimentally is the inverse problem of regenerating the interdiffusion coefficient from data on the dependence of the concentration on time and the coordinates. Further obstacles arise as a result of the limited accuracy of measurement of

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